

Grade 8
New Bedford Public Schools - Mathematics Curriculum Map for 2014-2015

Concept Focus Unit	Domains/Standards	Time
Unit 1: <u>Introducing Transformations</u>	SMP: 5,6 8.G.1 a-c	10 days
Unit 2: <u>Understanding Congruence Through Transformations</u>	SMP: 3,7 8.G.2,8.G.3	11 days
Unit 3: <u>Understanding Similarity</u>	SMP: 3,6 8.G3,8.G.4,8.G.5	13 days
Unit 4: <u>Rational and Irrational Numbers</u>	SMP: 1,6 8.NS.1, 8.NS.2, 8.EE.2	7 days
Unit 5: <u>Pythagorean Theorem</u>	SMP: 3,4,7,8 8.G6, 8.G7, 8.G8	13 days
Unit 6: <u>Functions</u>	SMP: 2,4,7 8.F.1,8.F.2,8.F.3,8.F.5	8 days
Unit 7: <u>Introduction to Linearity</u>	SMP: 4,8 8.EE.5, 8.EE.6, 8.F.4, 8.F.5	11 days
Unit 8: <u>Data Analysis</u>	SMP: 3,5 8.SP.1, 8.SP.2, 8.SP.3., 8.SP.4	15 days
Unit 9: <u>Nonlinear Functions</u>	SMP: 1,2 8.F.3, 8.F.5	11 days
Unit 10: <u>Solving Linear Equations</u>	SMP: 6,7 8.EE.7 a-b	17 days
Unit 11: <u>Systems of Linear Equations</u>	SMP: 1,6,7 8.EE.8 a-c	17 days
Unit 12: <u>Exponents and Scientific Notation</u>	SMP: 4,5,6,8 8.EE.1, 8.EE.3, 8.EE.4	11 days
Unit 13: <u>Angles - Geometric Relationships</u>	SMP: 3,5 8.G.5	11 days
Unit 14: <u>Volume - Cones, Spheres, and Cylinders</u>	SMP: 1,2 8.G.9	12 days
PARCC testing: Performance Based Assessment (PBA): March 16-April 10 computer-based; March 23 – April 3 paper-based End of Year assessment (EOY): May 4 – 29 computer-based; May 11 – 22 paper-based		
Review and Extend	All standards	May/June
Final Assessment	All standards	June

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This NBPS Math Curriculum map reflects the MA 2011 Mathematics Curriculum Frameworks and is aligned to the Common Core State Standards for mathematical content as well as the following eight standards of Mathematical practice.

Standards for Mathematical Practice (SMP)

- | | |
|---|---|
| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
| 3. Construct viable arguments and critique the reasoning of others. | 7. Look for and make use of structure. |
| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

The Standards for Mathematical Practice (SMP) will need to be an integral part of mathematical instruction. In grades 6-8, each instructional unit will highlight suggested standards of mathematical practice to focus on for that unit. Teachers are encouraged to use any standards of math practice that are appropriate for any given lesson but the assessment for that unit will incorporate the focus SMP for that unit. In this way, teachers will assure that each of the 8 SMP will be a focus in several units throughout the year.

WIDA English Language Development standard 3 - The Language of Mathematics

English language learners **communicate** information, ideas and concepts necessary for academic success in the content area of **Mathematics**.

Unit plans will include specific Performance Indicators (PI) and performance tasks and lesson plans will need to include specific language objectives appropriate to the content and ELD level of students.

Listed below are important and useful resources available on the Internet for teachers. Please add appropriate online resources as needed.

Website:	Notes:
www.ati-online.com (select Galileo K-12)	Each teacher will receive access information to the Galileo K-12 program, which will be used for district math assessments and intervention options. Common unit assessments, district benchmark assessments, assessment analysis reports, instructional dialogs (instructional supports), and student histories will be available to support standards-based instruction in mathematics.
http://katm.org/wp/?page_id=91	Grade level “flip books” with suggestions for integration of Standards for Math Practice for each standard, explanation of the content standard, instructional strategy recommendations, student misconceptions to address, etc. Developed with NC, Ohio, and Arizona departments of education.
www.parcconline.org	Implementation of the common core guidelines for gr. 3-8 and HS. This guide was used for reference as the district math curriculum maps were revised.
http://www.doe.mass.edu/frameworks/math/0311.pdf	Massachusetts 2011 Mathematics Curriculum Frameworks, which incorporate the Common Core State Standards for content and the standards of mathematical practice.
www.wida.us	WIDA standards for ELL students include specific standards for content areas such as math. This website has a downloadable library of resources for teachers.

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Unit 1: Introducing Transformations	Suggested number of days: 10
This unit provides an introduction to transformations as students explore the three congruence transformation: rotations, reflections, and translations. This unit builds on students' work in prior grades with parallel lines, angles, and symmetry in geometric figures (4G1, 4G3) and the coordinate plane (6NS6). In units 2 and 3 students build upon these foundations as they investigate combination of translations that result in congruence or similarity transformations. This work with congruence and similarity provides support for development of the formal definition of slope in unit 7.	
State Standards	Suggested Resources
Cluster Heading: Understand congruence and similarity using physical models, transparencies, or geometry software.	
8.G.1 - Verify experimentally the properties of rotations, reflections, and translations: 8.G.1.a - Lines are taken to lines, and line segments to line segments of the same length 8.G.1.b - Angles taken to angles of the same measure 8.G.1.c - Parallel lines are taken to parallel lines	CMP: <u>Common Core (CC) Investigation 3</u> Glencoe: Ch. 6 – Lessons 1-3 and Assessments CMP: <u>Kaleidoscopes, Hubcaps, and Mirrors</u> Invs. 1, 2, 3, 4, 5 PH: 3.8 – 3.10
<ul style="list-style-type: none"> Identify lines of reflection (symmetry) Relate symmetry and linear relationships (parallel to parallel) to reflected and rotated images. Verify the properties of rotations, reflections, and translations. 	

Standards for Mathematical Practice – Unit 1	
5. Use appropriate tools strategically.	As students investigate transformations, they attend to precision (SMP6) as they use appropriate terminology to describe and verify the properties of the various transformations. They also select and use tools such as geometry software, coordinate planes, and tracing paper strategically (SMP5).
6. Attend to precision.	

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Unit 2: Understanding Congruence through Transformations		Suggested number of days: 11
Comments:		
State Standards		Suggested Resources
Cluster Heading: Understand congruence and similarity using physical models, transparencies, or geometry software.		
8.G.2 - Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them		CMP: <u>Common Core (CC) Investigation 3</u> Glencoe: Ch. 6 – Lessons 7-1, 7-2 and Assessments CMP: <u>Kaleidoscopes, Hubcaps, and Mirrors</u> Inv. 2: ACE 24-25, 32, Inv. 3 and 5
<ul style="list-style-type: none"> • Identify congruent parts of congruent figures • Identify congruent figures • Apply understanding of congruence of angles and line segments to translated, reflected, and rotated images. 		
8.G.3 - Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.		Suggested websites: National Library of Virtual Manipulatives – http://nlvm.uwsu.edu PH: 5.6
<ul style="list-style-type: none"> • Use ratio and proportion in the solutions of problems involving similar plane figure. • Predict the results of transformations on figures 		

Standards for Mathematical Practice – Unit 2	
3. Construct viable arguments and critique the reasoning of others.	Students construct viable arguments and critique the reasoning of others (SMP3) as they describe the effect of transformation. As students investigate those effects, they attend to structure (SMP7) by recognizing the common attributes as properties generated by the transformations.
7. Look for and make use of structure.	

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Unit 3: Understanding Similarity		Suggested number of days: 13
In unit 13, students investigate the remaining geometric relationships described in 8.G.5		
State Standards Cluster Heading: Understand congruence and similarity using physical models, transparencies, or geometry software. 8.G.3 - Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. <ul style="list-style-type: none"> • Use ratio and proportion in the solutions of problems involving similar plane figure. • Predict the results of transformations on figures 8.G.4 - Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. <ul style="list-style-type: none"> • Identify the transformations applied from one similar figure to another, e.g., • Understand that congruent figures are also similar with a 1-1 ratio. 8.G.5 - Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. <i>For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</i> <ul style="list-style-type: none"> • Use concrete examples of triangles to show that when two angles are congruent, then the sides of one triangle are in proportion to the sides of the other triangle 	Suggested Resources Glencoe: Ch. 7 – Lessons 3,4,7; Problem solving Inv. 5 and Assessments CMP: CC Investigation 3, 4 CMP: Kaleidoscopes, Hubcaps, and Mirrors Inv. 2: ACE 24-25, 32, Inv. 5 Suggested websites: Gr. 8 Flipbook - http://katm.org/wp/?page_id=91 ; National Library of Virtual Manipulatives – http://nlvm.uwsu.edu - PH: 5.6, 8.1 – 8.2, 8.2 extension (page 419), 8.3	

Standards for Mathematical Practice – Unit 3	
3. Construct viable arguments and critique the reasoning of others.	As with unit 2, students attend to precision (SMP.6) as they construct viable arguments and critique the reasoning of others (SMP.3) while describing the effects of similarity transformations and the angle-angle criterion for similarity of triangles.
6. Attend to precision.	

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Unit 4: The Rational and Irrational Numbers	Suggested number of days: 7
This unit introduces the real number system and how real numbers are used in a variety of contexts. Students become familiar with irrational numbers (especially square and cube roots), but also learn how to solve equations of the form $x^2 = p$ and $x^3 = p$. Incorporating the Equations and Expression standards with the Number System standards provides context and motivation for learning about irrational numbers: for instance, to find the side length of a square of a certain area.	
<u>Please note:</u> Students will need to apply previous understandings of estimation, decimals, and fractions in this unit. Work with these skills needs to be embedded in the context of the number system.	
<ul style="list-style-type: none"> Translate between fractions and decimals in order to determine if a number is rational or irrational. Apply estimation skills to locate whole numbers, square roots, and decimals on a number line Use familiar fractions (quarters, halves, etc.) and their decimal approximations to estimate locations of fractions, mixed numbers, and decimals on a number line. 	
State Standards	Suggested Resources
Cluster Heading: Know that there are numbers that are not rational, and approximate them by rational numbers	
8.NS.1 - Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number	CMP: Common Core (CC) Investigation CMP: Looking for Pythagoras: Inv.2,3,4 CC Investigation 1 Glencoe: Ch. 1 – Lessons 1, 8,9,10 and Assessments Calculators suggested for calculating decimal expansions of irrational numbers
<ul style="list-style-type: none"> Explore real rational and irrational numbers and compare their decimal expansions. Identify numbers with non-repeating decimals as irrational, especially common irrational numbers such as $\sqrt{2}$ and π. Convert a repeating decimal expansion into a rational number, e.g., $2.333\dots = 2 \frac{1}{3}$ or $\frac{7}{3}$ 	PH: 4.2,4.3, 4.8, 6.1, 6.2
8.NS.2 - Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g. Pi). For example, by truncating the decimal expansion of square root of 2, show that square root of 2 is between 1 and 2, then between 1.4 and 1.5 and explain how to continue on to get better approximations	See Map Appendix: Illustration 1 – The Number system
<ul style="list-style-type: none"> Estimate and position decimals and decimal approximations for square roots and irrational numbers, such as pi, on the number line 	
8.EE.2 - Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. Also e.g. $(1/2)^3$ or $\sqrt{4/9}$	
<ul style="list-style-type: none"> Apply the rules of powers, roots, and cube roots to the solution of problems. Extend the order of operations to include positive integer exponents, square roots, and cube roots. 	

****See next page for the rest of unit 4**

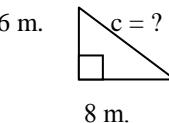
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Unit 4: Rational and Irrational Numbers continued:

Standards for Mathematical Practice – Unit 4	
1. Make sense of problems and persevere in solving them.	Understanding irrational numbers and their decimal approximations and evaluating square and cube roots requires persistence (SMP.1) with precision and estimation (SMP.6). Students look to express regularity and repeated reasoning as they convert fractions to decimals and notice that when they repeat the same calculations, the decimal also repeats (SMP.8).
6. Attend to precision.	
8. Look for and express regularity in repeated reasoning.	

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Unit 5: Pythagorean Theorem		Suggested number of days: 13
This unit provides further motivation and context for using square roots. In future math course, the Pythagorean Theorem will continue to play an important role.		
State Standards	Suggested Resources	
Cluster Heading: Understand and apply the Pythagorean Theorem		
8.G.6 - Explain a proof of the Pythagorean Theorem and its converse.	CMP: <u>Common Core (CC)</u> <u>Investigation</u>	
<ul style="list-style-type: none"> Demonstrate an understanding of the Pythagorean Theorem and it's converse both verbally and in diagrams 		
8.G.7 - Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions (including measurements and grids)	Glencoe: Ch. 5 – Lessons 2,5,6,7 and Assessments CMP: <u>Looking for Pythagoras</u> Investigations 2, 3, 4 PH: 4.9 extension (page 232)	
<ul style="list-style-type: none"> Use the relationship between squaring and finding the square root to solve Pythagorean theorem problems Identify the Pythagorean formula Identify the legs and hypotenuse and how they relate to the equation Determine if 3 line segments of a certain length would create a right triangle Apply the Pythagorean Theorem to find the length of a side of a triangle in a real world situation, e.g., In order to find the perimeter of a right triangle, use the Pythagorean Theorem to find the third side of a triangle given the height and base of the triangle. <p>6 m.  $6^2 + 8^2 = c^2$ 8 m. $36 + 64 = c^2$ $100 = c^2$ $\sqrt{100} = \sqrt{c^2}$ $10 = c$ Therefore the perimeter is $10 + 6 + 8 = 24$ m.</p>	Suggested websites: National Library of Virtual Manipulatives – http://nlvm.uwsu.edu - Geoboard – coordinate, Geoboard,	
8.G.8 - Apply the Pythagorean Theorem to find the distance between two points in a coordinate system		
<ul style="list-style-type: none"> Identify a right triangle within another figure. Know that the diagonal of a unit square is $\sqrt{2}$ Apply the Pythagorean Theorem to find the length of a diagonal line on a coordinate grid. Apply the Pythagorean Theorem to find the lengths of the sides of a triangle drawn on a coordinate grid. 		
Standards for Mathematical Practice - Unit 5	Understanding, modeling, and applying (SMP4) the Pythagorean theorem and its converse require that students look for and make use of structure (SMP7) and express repeated reasoning (SMP8). Students also construct and critique arguments as they explain a proof of the Pythagorean Theorem and its converse (SMP3).	
3. Construct viable arguments and critique the reasoning of others.		
4. Model with mathematics.		
7. Look for and make use of structure.		
8. Look for and express regularity in repeated reasoning.		

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Unit 6: Functions	Suggested number of days: 8								
In this unit, the term function is formally introduced for both linear and non-linear functions. Students model functions in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions) and interpret those representations qualitatively and quantitatively.									
<ul style="list-style-type: none"> • 8.F.A.3 Students will more fully investigate non-linear functions in unit 9. For this unit, students will be introduced to non-linear functions so that they are able to identify whether or not a function is linear, but they will not be required to generate their own examples until unit 9. • 8.F.B.5 will also be addressed in unit as students investigate non-linear functions more explicitly. 									
State Standards	Suggested Resources								
<i>Cluster Heading: Define, evaluate, and compare functions</i>									
8.F.1 - Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.¹ Function notation is not required in Grade 8)	<p>CMP: <u>Common Core (CC) Investigation 2</u></p> <p>**Glencoe: Ch. 3-Lessons 2-4; Ch. 4 – Lessons 1,2,3,5, and Assessments</p> <p>CMP: <u>Thinking with Mathematical Models</u> - Investigation 1</p> <p>CMP: <u>Growing, Growing, Growing</u> Investigation 1 and ACE 25-26, 38, 47</p> <p>CMP: <u>Frogs, Fleas, and Painted Cubes</u> Investigations 2, 3, 4</p> <p>CMP: <u>Say It With Symbols</u> Investigation 2</p> <p>PH: 3.1, 3.2, 12.2-12.4</p> <p>See Map Appendix: Algebra Four-block template</p> <p>Suggested websites: National Library of Virtual Manipulatives – http://nlvm.uwsu.edu - Grapher</p>								
<ul style="list-style-type: none"> • Describe and write sequence patterns • Extend and analyze patterns using graphs, words, tables, symbolic expressions • Extend and analyze arithmetic and geometric progressions using graphs, words, tables, symbolic patterns • Use process (function) diagrams, input/output tables, and words to illustrate the output as a function of the independent variable, e.g. $x \rightarrow \boxed{+3} \rightarrow y$ or “add three to x to get y” or “take the input and add three to get the output”, or <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <th style="text-align: center;">Input</th><th style="text-align: center;">Output</th></tr> <tr> <td style="text-align: center;">1</td><td style="text-align: center;">4</td></tr> <tr> <td style="text-align: center;">2</td><td style="text-align: center;">5</td></tr> <tr> <td style="text-align: center;">x</td><td style="text-align: center;">$x + 3$</td></tr> </table> <p>Generalize the above patterns – find a “rule”</p>	Input	Output	1	4	2	5	x	$x + 3$	
Input	Output								
1	4								
2	5								
x	$x + 3$								
8.F.2 - Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.									
<ul style="list-style-type: none"> • Given a table, identify the pattern of change in the y values • Given a table identify the pattern of change in the x values • Use the patterns of change in the y and x to find the slope • Given a table, identify the rule for the table e.g., equation of the function 									

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Unit 6: Functions - continued		Suggested numbers of days: 8		
In this unit, the term function is formally introduced for both linear and non-linear functions. Students model functions in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions) and interpret those representations qualitatively and quantitatively.				
State Standards	Suggested Resources			
<i>Cluster Heading: Define, evaluate, and compare functions</i>				
8.F.3 - Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear				
<ul style="list-style-type: none"> Apply the ability to identify and graph points in a coordinate plane to graph equations with two variables (linear and non-linear) Identify the slope of a vertical line as undefined and the slope of a horizontal line as 0, (0 rate change) Given a line identify its slope (rate of change) as negative (decreasing) or positive (increasing) Given a line, write the equation in slope-intercept form Given an equation ($y = mx + b$) identify slope (rate of change) and y-intercept (constant, fee, etc.) Graph equations with two variables by using tables and/or slope and y-intercepts. Identify functions that are not linear, e.g., $y = x^2$ or graph without a constant rate of change. 	** See Glencoe resources listed above PH: 3.3 – 3.4, 12.5, 12.6 CMP: <u>Thinking with Mathematical Models</u> : Inv. 2, 3 <u>Growing, Growing, Growing</u> : Inv. 1, 2, 3, 4 <u>The Shapes of Algebra</u> : Inv. 4 <u>Say It With Symbols</u> : Inv. 4 <u>Frogs, Fleas, and Painted Cubes</u> : Investigation 1, 2, 3, 4 <u>Say It With Symbols</u> : Inv. 4			
<i>Cluster Heading: Use functions to model relationships between quantities</i>				
8.F.5 - Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.				
Use linear equation and properties to model and analyze problems Create and use symbolic expressions and relate them to verbal, tabular, and graphical representations for linear relationships				

Standards for Mathematical Practice – Unit 6	
2. Reason abstractly and quantitatively.	Understanding how functions model (SMP4) relationships requires that students reason abstractly and quantitatively (SMP.2) while looking for and making use of structure (SMP7).
4. Model with mathematics.	
7. Look for and make use of structure.	

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Unit 7: Introduction to Linearity		Suggested number of days: 11
This unit builds on students' understanding of function and their work in earlier grades on constant of proportionality (7.RP.A.2) as they formally investigate linear relationships and slope. Students construct functions to model linear relationships.		
State Standards	Suggested Resources	
Cluster Heading: Understand the connections between proportional relationships, lines, and linear equations		
8.EE.5 - Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed	CMP: Common Core (CC) <u>Investigations:</u> Inv. 2 Glencoe: Ch. 3 – Lesson 1, 6; Ch. 4 Lessons 1,3,4,7,8,9, and Assessments PH: 3.3 (include extension page), 3.4, 5.1, 5.3, 5.4, 12.5	
<ul style="list-style-type: none"> Find unit rates and scale factors given a context Set up correct proportions for the problem and solve them Use models, graphs and formulas to solve simplify problems involving velocity and density 	CMP: Thinking with Mathematical Models: Investigations 1, 2, 3, <u>The Shapes of Algebra:</u> Inv. 4 <u>Say It With Symbols:</u> Inv. 4 <u>Growing, Growing, Growing:</u> Investigations 1, 2, 3, 4 <u>Frogs, Fleas, and Painted Cubes:</u> Investigation 1, 2, 3, 4	
8.EE.6 - Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	<ul style="list-style-type: none"> Identify the roles of variables within an equation, $y = mx + b$, expressing y as a function of x with parameters m and b. Classify figures in terms on congruence and similarity, relating slope and similar triangles. 	
Cluster Heading: Use functions to model relationships between quantities		
8.F.4 - Construct a function to model a relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values	<ul style="list-style-type: none"> Identify the relationships of variables from a variety of displays – verbal, graph, table, and equation. Apply the concept of slope (rate of change) to equations that model real life applications Set up and solve linear equations and inequalities Explain and analyze, both quantitatively and qualitatively, using pictures graphs, charts, and equations Use models graphs and formulas to solve simple problems involving rates and velocity Translate between table, equation, graph, and verbal rule to solve problems (Algebra four-block template) Find the slope of a line from a graph, context, table, and given two coordinates (change in y /change in x) 	
8.F.5 - Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	<p>Suggested websites: National Library of Virtual Manipulatives – http://nlvm.uwsu.edu - Grapher</p> <ul style="list-style-type: none"> See Map Appendix: Algebra Four-block template 	
<ul style="list-style-type: none"> Use linear equation and properties to model and analyze problems Create and use symbolic expressions and relate them to verbal, tabular, and graphical representations for linear relationships 		

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Unit 7: Introduction to Linearity continued...

Standards for Mathematical Practice – Unit 7	
4. Model with mathematics.	Constructing functions to model linear relationships (SMP4) requires that students look for and express regularity in repeated reasoning (SMP8).
8. Look for and express regularity in repeated reasoning.	

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Unit 8: Data Analysis	Suggested number of days: 15
In this unit, students will investigate bivariate categorical and numerical data. The work with categorical data connects with students' prior work with proportional relationships and rational numbers; the work with numerical data builds on students' learning from earlier units around linear functions and modeling.	
State Standards	Suggested Resources
<i>Cluster Heading: Investigate patterns of association in bivariate data.</i>	
<p>8.SP.1 - Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. e.g. as the number of hours spent studying increases the grade percentages increased.</p> <ul style="list-style-type: none"> • Create and analyze line plots, scatter plots, Venn diagrams, stem and leaf plots, box and whisker plots, histograms and circle graphs 	<p>CMP: <u>Common Core (CC)</u> <u>Investigation - Investigation 5</u></p> <p>Glencoe: Ch. 9 – Lessons 1-3 and Assessments</p>
<p>8.SP.2 - Know that straight lines are widely used to model relationships between quantitative variables. For scatter plots that suggest a linear association, informally assess the model fit by judging the closeness of the data points to the line.</p> <ul style="list-style-type: none"> • Display data using intervals • Create and analyze line plots, scatter plots, Venn diagrams, stem and leaf plots, box and whisker plots, histograms and circle graphs 	<p>CMP: <u>Samples & Populations</u> <u>Investigation 4</u> <u>Thinking With Mathematical Models</u> <u>Investigation 2,3</u></p> <p>CMP: <u>Shapes of Algebra</u> <u>Investigations 2, 3</u></p>
<p>8.SP.3 - Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.</p> <ul style="list-style-type: none"> • Interpret the roles of variables within $y = mx + b$ within context of a word problem Use linear equations to model and analyze problems involving proportional relationships 	<p>PH: 10.1 – 10.2, 10.3,10.5</p>
<p>8.SP.4 - Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</p> <ul style="list-style-type: none"> • Select, create, interpret and utilize various tabular and graphical representations of problems. • Specify frequencies and relative frequencies 	

Standards for Mathematical Practice – Unit 8	Representing and analyzing data requires that students use appropriate tools strategically (SMP5) and construct and critique arguments (SMP3) about the data.
3. Construct viable arguments and critique the reasoning of others.	
5. Use appropriate tools strategically.	

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Unit 9: Nonlinear functions	Suggested numbers of days: 10			
State Standards	Suggested Resources			
<i>Cluster Heading: Define, evaluate, and compare functions</i>				
8.F.3 - Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.				
<ul style="list-style-type: none"> Apply the ability to identify and graph points in a coordinate plane to graph equations with two variables (linear and non-linear) Identify the slope of a vertical line as undefined and the slope of a horizontal line as 0, (0 rate change) Given a line identify its slope (rate of change) as negative (decreasing) or positive (increasing) Given a line, write the equation in slope-intercept form Given an equation ($y = mx + b$) identify slope (rate of change) and y-intercept (constant, fee, etc.) Graph equations with two variables by using tables and/or slope and y-intercepts. Identify functions that are not linear, e.g., $y = x^2$ or graph without a constant rate of change. 	CMP: <u>Common Core (CC) Investigation</u> Glencoe: Ch. 4 – Lessons 7 and 8; and Assessments PH: 3.3 – 3.4, 12.5, 12.6 <ul style="list-style-type: none"> CMP: <u>Thinking with Mathematical Models</u>: Inv. 2, 3 <u>Growing, Growing, Growing:</u> Inv. 1, 2, 3, 4 <u>The Shapes of Algebra:</u> Inv. 4 <u>Say It With Symbols:</u> Inv. 4 <u>Frogs, Fleas, and Painted Cubes:</u> Investigation 1, 2, 3, 4 			
<i>Cluster Heading: Use functions to model relationships between quantities</i>				
8.F.5 - Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.				
<ul style="list-style-type: none"> Use linear equation and properties to model and analyze problems Create and use symbolic expressions and relate them to verbal, tabular, and graphical representations for linear relationships 				

Standards for Mathematical Practice – Unit 9	
1. Make sense of problems and persevere in solving them.	As students sketch graphs that exhibit the qualitative features of functions, they make sense of the situations (SMP.1) while reasoning abstractly and quantitatively (SMP.2).
2. Reason abstractly and quantitatively.	

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Unit 10: Solving linear equations		Suggested number of days: 11
With this unit, students build on their prior work in writing and solving equations (6.EE.B.7, 7.EE.B.4a) and their understanding of algebraic representations of linear functions.		
State Standards	Suggested Resources	
<i>Cluster Heading: Analyze and solve linear equations...</i>		
8.EE.7 Solve linear equations in one variable. 8.EE.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). 8.EE.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	CMP: <u>Common Core (CC) Investigation:</u> Investigation 2 Glencoe: Ch. 2 – Lessons and Assessments PH: 1.8, 2.1 – 2.5, 3.2, 12.8 – 12.10 CMP: <u>Thinking With Mathematical Models:</u> Investigation 2 <u>Say It With Symbols:</u> Inv.1, 2, 3	
<ul style="list-style-type: none"> Write equations that would have no solutions ($a = b$), e.g., $x + 3 = x + 4$ ($a = b$), one solution, e.g., $2x = 6$ ($x = a$), or infinite solutions ($a = a$), e.g., $2(x+1) = 2x + 2$ Identify and use properties to simplify and compute. Correctly simplify an expression using distributive property (include problems with variables) 	See Map Appendix: Table 4 – Property of Equality	
Standards for Mathematical Practice – Unit 10	Writing and solving equations require that students make use of structure (SMP7) and attend to precision (SMP.6) as students apply properties of operations to transform equations into simpler forms.	
6. Attend to precision		
7. Look for and make use of structure.		

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Unit 11: Systems of Linear Equations		Suggested number of days: 17
This unit extends student' facility with solving problems by writing and solving equations.		
State Standards	Suggested Resources	
<i>Cluster Heading: Analyze and solve linear equations and pairs of simultaneous linear equations</i>		
8.EE8 Analyze and solve pairs of simultaneous linear equations	CMP: Common Core (CC) Investigation	
<p>8.EE.8a - Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</p> <p>8.EE.8b - Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. <i>For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</i></p> <p>8.EE.8c - Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair</p>	<p>Glencoe: Ch. 3 – Lessons 7 and 8, and Assessments</p> <p>PH: 3.6 - 3.7</p> <p>CMP:The Shapes of Algebra: Investigations 1- ACE 56-57, 2, 3, 4</p>	
<ul style="list-style-type: none"> Estimate the coordinate point which satisfies both equations from a graph Given two pairs of points, use graphing or algebra to determine if the lines intersect Analyze and solve pairs of simultaneous linear equations algebraically: setting two equations ($y = mx + b$ form) equal, substitution, or elimination methods Identify the solution (x,y values) and confirm the solution in both equations <p>Solve real-world and mathematical problems which lead to two linear equations in two variables, e.g., compare two cell phone plans to see which company is a better buy for 100 minutes a month – Company A - \$.25/minute or Company B - \$10 monthly fee and \$.05/minute</p>	<p>Suggested websites: National Library of Virtual Manipulatives – http://nlvm.uwsu.edu - Grapher</p>	

Standards for Mathematical Practice – Unit 11	
1. Make sense of problems and persevere in solving them.	Students' perseverance in solving real-world problems with systems of equations requires that they work with various solution methods and learn to discern when each method is most appropriate (SMP.1). As with the previous unit, writing and solving systems require that students make use of structure (SMP.7) and attend to precision (SMP.6) as students apply properties of operations to transform equations into simpler forms.
6. Attend to precision.	
7. Look for and make use of structure.	

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Unit 12: Exponents and Scientific Notation	Suggested numbers of days: 11
This unit builds on the work of unit 4 as students extend their understanding of radicals and integer exponents to develop rules for working with exponents and scientific notation. Although the standards in this unit are in a major content cluster for Grade 8, this unit is sequenced near the end of the year since the content of this unit is not explicitly connected with other major content at this grade; however, this unit could be placed earlier in the year if desired.	
State Standards	Suggested Resources
Cluster Heading: Work with radicals and integer exponents 8.EE.1 - Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$</i> <ul style="list-style-type: none"> Apply the rules of powers and roots to the solution of problems, Extend the Order of Operations to include positive integer exponents and square roots. Use the understanding that $x^0 = 1$ and $x^1=x$ to simplify expressions Use positive and negative exponents and standard notation and include more complicated numbers such as $256,703 = 2.56703 \times 10^5$ 8.EE.3 - Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. <i>For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.</i> <ul style="list-style-type: none"> Represent numbers in scientific notation and use them in real world problem solving Use positive and negative exponents and standard notation and include more complicated numbers such as $256,703 = 2.56703 \times 10^5$ 8.EE.4 - Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. <ul style="list-style-type: none"> Select convert and use appropriate units of measure or scale. Represent numbers in scientific notation and use them in calculations and problem solving situations, including the four operations 	CMP: Common Core (CC) Investigation – Investigation 1 Glencoe: Ch. 1 – Lessons 2, 3,4,6,7 and Assessments PH: 1.7, 7.1- 7.5 CMP: Growing, Growing, Growing. Inv. 1: ACE 39-40 Inv. 2: ACE 15-17: Inv. 4: ACE 8 Inv. 5: ACE 56-60 See Map Appendix: Table 4 Properties of Operations

Standards for Mathematical Practice – Unit 12	
4. Model with mathematics.	Modeling mathematics (SMP.4) with radicals, integer exponents, and scientific notation requires that students attend to precision (SMP6) and look for and express regularity in repeated reasoning (SMP.8). Students will also need to use appropriate tools strategically since some calculation can be completed more easily through visual inspection than with a
5. Use appropriate tools strategically.	
6. Attend to precision.	

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8. Look for and express regularity in repeated reasoning.	calculator (SMP5).	
Unit 13: Angles – geometric relationships		Suggested number of days: 11
This unit builds on students' work in Grade 7 with angle relationships (7.G.2, 7.G.5) as they work with angle relationships in triangles and investigate parallel lines cut by transversal.		
Comments:		
<ul style="list-style-type: none"> • 8.A.5 The angle-angle criterion for similarity of triangles was investigated in unit 3. 		State Standards
<i>Cluster Heading: Understand congruence and similarity using physical models, transparencies, or geometry software.</i>		Suggested Resources
8.G.5 - Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.		CMP: Common Core (CC) Investigation - Investigation 4 Glencoe: Ch. 5 – Lessons 1,3,5 and Assessments PH: 8.1 – 8.2, 8.2 extension (page 419) Suggested websites: Gr. 8 Flipbook - http://katm.org/wp/?page_id=91
<ul style="list-style-type: none"> • Use and identify vertical, adjacent, supplementary, complementary angles, alternate interior angles and corresponding angles • Identify parallel lines, perpendicular lines and transversals • Find the complement and supplement of a given angle • Find and identify the measures of angles formed by parallel lines • Classify triangles and quadrilaterals • Distinguish between regular and irregular polygons • Calculate the sum of the interior angles of a polygon by dividing the shape in to triangles and using $(n-2)180$ where n is the number of sides of the polygon • Derive the formula $(n-2)180$ • Calculate the number of degrees in <u>one</u> angle of a regular polygon • Use the relationship between the sides and the sums of the interior and exterior angle measures of polygons to find the measure of angles and number of sides 		

Standards for Mathematical Practice – Unit 13	Students use tools strategically (SMP5) as they investigate angle relationships and generate or critique informal arguments (SMP3) to establish facts about angle relationships.
3. Construct viable arguments and critique the reasoning of others.	
5. Use appropriate tools strategically.	

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Unit 14: Volume – Cones, spheres, and cylinders		Suggested number of days: 12
In Grade 7, students solved real-world problems involving surface area and volume of prisms and pyramids. In this unit, those understandings are extended to include problems solving with cylinders, cones, and spheres.		
State Standards	Suggested Resources	
<i>Cluster Heading:</i> Solve real-world and mathematical problems involving volumes of cylinders, cones, and spheres		
8.G.9 - Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	CMP: Common Core (CC) Investigation -Investigation 4 Glencoe: Ch. 8 – Lessons 1,2,3 (surface area extension Lessons 4,5) and Assessments PH: 9.6-9.9, 9.7 extension (page 511) CMP: <u>Kaleidoscopes, Hubcaps, and Mirrors</u> Inv. 1: ACE 47-49 Inv. 2: ACE 28 Inv. 3: ACE 24 <u>Looking for Pythagoras.</u> Inv. 3: ACE 18-22, 25-26 Inv. 4: ACE 57-58 <u>Say It With Symbols</u> Inv. 1: ACE 55 Inv. 3: ACE 41 Inv. 4: ACE 39	
<ul style="list-style-type: none"> Demonstrate an understanding of area as square units of measure Find the area and perimeter of parallelograms (all types), trapezoids, irregular shapes or shaded regions. Identify the radius and diameter of a circle, or the base(s) and heights of a figure. Use π to calculate area of circles Manipulate a formula to solve for an unknown, Demonstrate an understanding of area and apply this to problems involving volume Find the volume 		

Standards for Mathematical Practice – Unit 14	
1. Make sense of problems and persevere in solving them.	As students model geometric relationships with formulas to solve problems (SMP.1) involving 3-dimensional figures, they reason both abstractly and quantitatively (SMP.2).
2. Reason abstractly and quantitatively.	

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Domains: ALL

PARCC Testing

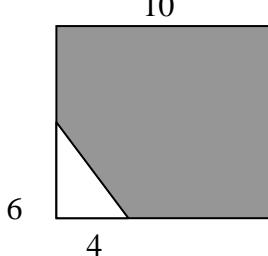
Mathematics Grade 8

Performance Based Assessment (PBA): March 16-April 10 computer-based, March 23 – April 3 paper-based

End of Year assessment (EOY): May 4 – 29 – computer-based, May 11 – 22 paper-based

REVIEW AND EXTEND 8TH GRADE CURRICULUM

The Review and Extend portion of the map is a time for you to review or re-teach areas of concern as needed based on assessment data from unit tests and benchmark assessments or to extend your students' learning to a higher level. The following are suggested activities that may help your planning and include materials that you have or may have access to. You may use additional appropriate resources that you find helpful. Please take the time to note additional resources for yourself and to share with others.

Prepare for final exam in June		
All domains and standards	Review and Extend Curriculum as needed based on assessment data from unit tests and benchmark assessments.	PH, Selected CMP Units
Pythagorean Theorem	<ul style="list-style-type: none">❖ Extend student understanding of the relationship between the distance formula and the Pythagorean Theorem❖ Calculate areas of figures using positive and negative space. For example: What is the area of the shaded portion of this figure? (note: 10 in square) 	Looking for Pythagoras
Other:		

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REVIEW AND EXTEND 8TH GRADE CURRICULUM

The Review and Extend portion of the map is a time for you to review or re-teach areas of concern as needed based on assessment data from unit tests and benchmark assessments or to extend your students' learning to a higher level. The following are suggested activities that may help your planning and include materials that you have or may have access to. You may use additional appropriate resources that you find helpful. Please take the time to note additional resources for yourself and to share with others.

Prepare for final exam in June	
All domains and standards	Review and Extend Curriculum as needed based on assessment data from unit tests and benchmark assessments.
Other:	
Other:	
Other:	

JUNE

FINAL ASSESSMENTS

Final District Math Benchmark Assessment

GRADE 8 MATH CURRICULUM MAP

APPENDIX

FROM MA. MATH 2011 CURRICULUM:

- TABLE 1 – COMMON ADDITION AND SUBTRACTION SITUATIONS
- TABLE 2 – COMMON MULTIPLICATION AND DIVISION SITUATIONS
- TABLE 3 – PROPERTIES OF OPERATIONS
- TABLE 4 – PROPERTIES OF EQUALITY
- TABLE 5 – PROPERTIES OF INEQUALITY
- ILLUSTRATION 1 – THE NUMBER SYSTEM
- ONLINE RESOURCES

ADDITIONAL RESOURCE:

- ALGEBRA FOUR-BLOCK TEMPLATE

TABLE 1. Common addition and subtraction situations.¹

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown²
Put Together/ Take Apart³	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare⁴	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

¹ Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

² These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

³ Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

⁴ For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

TABLE 2. Common multiplication and division situations.⁵

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays,⁶ Area⁷	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Compare	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

⁵ The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

⁶ The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

⁷ Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

TABLE 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

<i>Associative property of addition</i>	$(a + b) + c = a + (b + c)$
<i>Commutative property of addition</i>	$a + b = b + a$
<i>Additive identity property of 0</i>	$a + 0 = 0 + a = a$
<i>Existence of additive inverses</i>	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
<i>Associative property of multiplication</i>	$(a \times b) \times c = a \times (b \times c)$
<i>Commutative property of multiplication</i>	$a \times b = b \times a$
<i>Multiplicative identity property of 1</i>	$a \times 1 = 1 \times a = a$
<i>Existence of multiplicative inverses</i>	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
<i>Distributive property of multiplication over addition</i>	$a \times (b + c) = a \times b + a \times c$

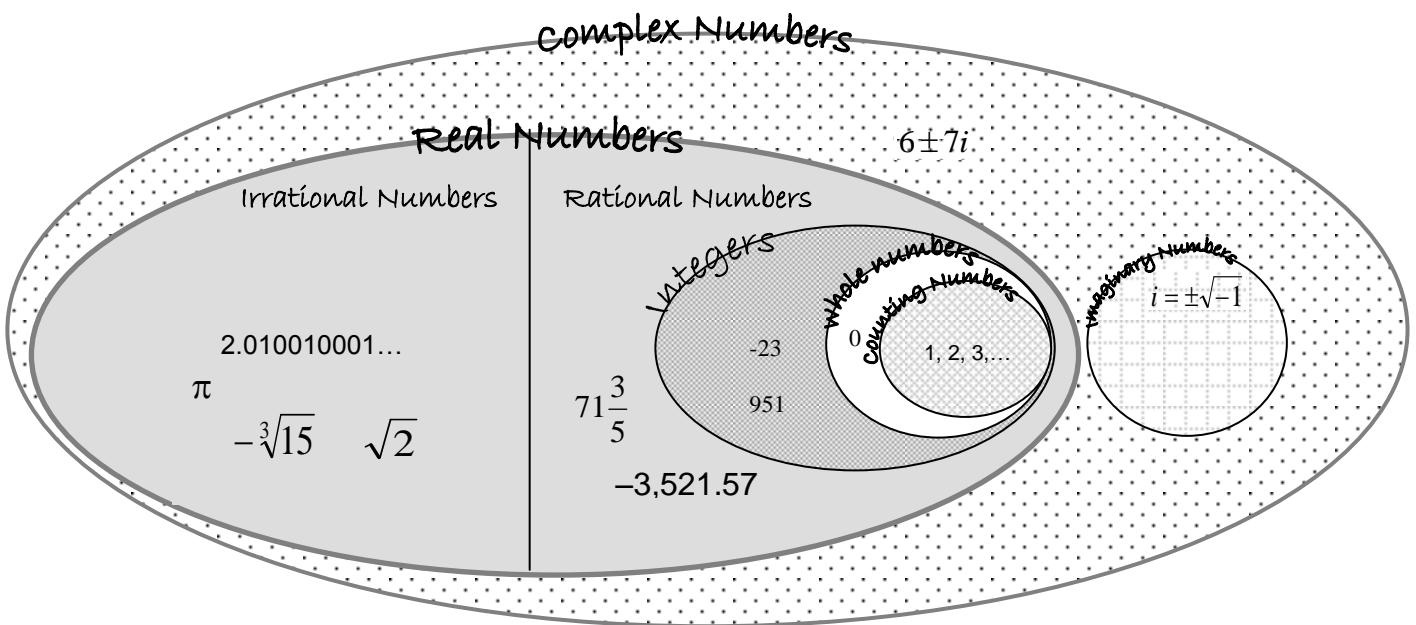
TABLE 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

<i>Reflexive property of equality</i>	$a = a$
<i>Symmetric property of equality</i>	If $a = b$, then $b = a$.
<i>Transitive property of equality</i>	If $a = b$ and $b = c$, then $a = c$.
<i>Addition property of equality</i>	If $a = b$, then $a + c = b + c$.
<i>Subtraction property of equality</i>	If $a = b$, then $a - c = b - c$.
<i>Multiplication property of equality</i>	If $a = b$, then $a \times c = b \times c$.
<i>Division property of equality</i>	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
<i>Substitution property of equality</i>	If $a = b$, then b may be substituted for a in any expression containing a .

TABLE 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

ILLUSTRATION 1. The Number System.



Algebra Four-block Template

verbal explanation/process diagram	Table
Equation	Graph